NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Memorandum 33-399

Revision I

Application of Structural Analysis and Matrix Interpretive System

R. M. Bamford

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CALIFORNIA INSTITUTE OF TECHNOLOGY
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Preface

The work described in this report was performed by the Engineering Mechanics Division of the Jet Propulsion Laboratory. Revision I was initiated to update time estimates. These time estimate changes are reflected in Fig. 1, Page 7. Also, various typographical errors have been corrected.

Acknowledgment

This report would not have been possible without the prior development of the Structural Analysis and Matrix Interpretive System by R. Melosh and others at Philco Western Development Laboratory and by J. Chisholm, T. Lang, L. Schmele, and V. Smith of JPL.

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Abstract

The matrix manipulations required for common structural analysis problems are developed, the control cards that direct the Structural Analysis and Matrix Interpretive System to perform these operations are listed, the required data are described, and a sample problem is given.

Application of Structural Analysis and Matrix Interpretive System

I. Introduction

A. Purpose and Orientation

This report provides for the utilization of the Structural Analysis and Matrix Interpretive System (SAMIS) (Refs. 1–3) in three ways. First, the report may be used as a training aid for analysts interested in learning to use SAMIS. Second, the report will allow the use of SAMIS on typical structural analysis problems without the analyst being overwhelmed by the intricacies and generalities of SAMIS. For this second use, only Sections III and IV are of interest. Third, the use of the standard set of control cards (listed in this report) on problems within their limitations will eliminate the time-consuming effort and errors typically encountered in setting up special, efficient sets for SAMIS.

It is the intention of the author to revise this report whenever radical changes are made in SAMIS which would allow either greater efficiency with a modified set of control cards or simpler input.

B. Type of Problem Solved

SAMIS, in conjunction with the listed control cards and the required data, will perform linear static analysis and calculate normal modes of most frame structures.

Displacements, reactions, frequencies, generalized weights, generalized stiffnesses, and member internal forces are generated for structures loaded by concentrated loads, reaction displacement, loads equal to the product of gridpoint weights and accelerations in g defined by the acceleration of the restraints, or undamped normal modes. For simplicity, only line elements are considered in this report.

In addition to the problems described, many other options of the basic SAMIS program can be used with the listed control cards. These options will be listed in Section V, but the details are not included since the extensions are not within the scope of this report.

II. Mathematics

A. Method

The method used is generally referred to as the stiffness method. Numerical manipulations are performed by SAMIS under the control of pseudoinstructions (control cards directing subprograms of SAMIS to perform specific operations) listed in this report. The pseudoinstructions, with one exception, represent operations performed on matrices. The exception is the structural modeling pseudoinstruction, which generates stiffness and internal force coefficient matrices from geometric and material properties of structural elements. The initial matrices defining the mathematical model of the structure and the boundary conditions are inputs or are generated by the structural modeling pseudoinstruction.

B. Development of Method

The method of computation for static or pseudostatic analysis and generation of normal modes will be developed in matrix notation. Notation is defined in Table 1 or within the text.

1. Static and pseudostatic analysis. The equilibrium equation in matrix notation is:

$$\mathbf{k}\,\mathbf{u} = \mathbf{f}$$

This equation relating stiffness $\mathbf{k}$, displacement $\mathbf{u}$, and force $\mathbf{f}$, is equally applicable to the equilibrium of a single structural element or the composite of all the elements. The stiffness matrix of the composite is the sum of the elemental stiffness matrices $\mathbf{k}_i$ generated by the structural modeling link and a stiffness matrix $\mathbf{k}_i$ that may be used to model elements other than those modeled by SAMIS. For compatibility of matrices, one may imagine the elemental matrices to have all the rows and columns of the composite matrix with zero elements, except in the rows and columns associated with the end gridpoints. For economy and to perform nonconformable matrix operations, SAMIS actually operates on and stores only nonzero elements and their codes (location within the matrix).

The displacement vector can be partitioned into reaction displacements $\mathbf{u}_R$ and unknown displacements $\mathbf{u}_U$. The stiffness matrix and force vector have corresponding partitions and the equilibrium equation can be rewritten in partitioned form as

$$\left[\begin{array}{c|c} \mathbf{k}_{UU} & \mathbf{k}_{UR} \\ \hline \mathbf{k}_{RU} & \mathbf{k}_{RR} \end{array}\right] \left\{\begin{array}{c} \mathbf{u}_U \\ \hline \mathbf{u}_R \end{array}\right\} = \left\{\begin{array}{c} \mathbf{f}_U \\ \hline \mathbf{f}_R \end{array}\right\}$$

If $\mathbf{f}_U = \mathbf{0}$ and $\mathbf{u}_R$ is consistent with rigid body displacement, then $\{\mathbf{u}_U\}$ is the rigid body displacement of the unrestrained degrees of freedom.

Omitting $\mathbf{f}_U$ and twice differentiating both sides of the equation with respect to time, the following equation is obtained:

$$[\mathbf{k}_{UU} \mid \mathbf{k}_{UR}] \left\{\begin{array}{c} \ddot{\mathbf{u}}_U \\ \hline \ddot{\mathbf{u}}_R \end{array}\right\} = \mathbf{0}$$

The acceleration field $\ddot{\mathbf{u}}_U$ is associated with a set of reaction accelerations $\ddot{\mathbf{u}}_R$, and $\ddot{\mathbf{u}}_U$ will be a rigid body acceleration if $\ddot{\mathbf{u}}_R$ is consistent with a rigid body displacement.

A load $\mathbf{f}_{UP}$ can be generated which is this acceleration $\ddot{\mathbf{u}}_U$ in g times the weight $\mathbf{w}_{UU}$ associated with each degree of freedom. This load is called a pseudostatic load since the resulting displacements are evaluated as for static loads, while the loading is associated with an acceleration field defined by the acceleration of the restraints.

The following equation allows evaluation of displacement $\mathbf{u}_{UP}$ due to pseudostatic loads:

$$\mathbf{k}_{UU}\,\mathbf{u}_{UP} = \mathbf{w}_{UU}\,\ddot{\mathbf{u}}_U\,\frac{1}{g}$$

Several load vectors may be simultaneously used in the same analysis. Separate displacement vectors with column codes corresponding to the column code of each load vector are generated. If a static load and an acceleration associated with a pseudostatic load have common column codes, the resulting displacements will be added, forming a single displacement vector. The displacements are computed from the following equation:

$$[\mathbf{k}_{UU} \mid \mathbf{k}_{UR}] \left[\begin{array}{c} \mathbf{u}_U \\ \hline \mathbf{u}_R \end{array}\right] = \left[\mathbf{f}_U + \mathbf{w}_{UU}\,\ddot{\mathbf{u}}_U\,\frac{1}{g}\right] \qquad (1)$$

Notice that $\ddot{\mathbf{u}}_U \dfrac{1}{g}$ is in units of the acceleration of gravity g, since $\ddot{\mathbf{u}}_R \dfrac{1}{g}$ is given in g.

2. Normal modes. The equation of equilibrium, including lumped mass and damping is:

$$\frac{1}{g}\,\mathbf{w}\,\ddot{\mathbf{u}} + \mathbf{C}\,\dot{\mathbf{u}} + \mathbf{k}\,\mathbf{u} = \mathbf{f}$$

Neither $\mathbf{C}$ nor $\mathbf{f}$ enter into the undamped eigenvalue problem, and only the unrestrained degrees of freedom of $\mathbf{k}$ and $\mathbf{w}$ are required, thus

$$\frac{1}{g}\,\mathbf{w}_{UU}\,\ddot{\mathbf{u}}_U + \mathbf{k}_{UU}\,\mathbf{u}_U = \mathbf{0}$$

Both $\mathbf{w}_{UU}$ and $\mathbf{k}_{UU}$ must be symmetric and positive definite, but $\mathbf{w}_{UU}$ is not required to be the same order as $\mathbf{k}_{UU}$. Multiplying both sides by $\mathbf{k}_{UU}^{-1}$ results in:

$$\frac{1}{g}\,\mathbf{k}_{UU}^{-1}\,\mathbf{w}_{UU}\,\ddot{\mathbf{u}}_U + \mathbf{I}\,\mathbf{u}_U = \mathbf{0} \qquad (2)$$

No inversion is actually performed; in practice the appropriate set of simultaneous equations is solved.

There is a unique lower triangular matrix $\mathbf{L}$ such that $\mathbf{L}\,\mathbf{L}^T = \mathbf{w}_{UU}$, since $\mathbf{w}_{UU}$ is positive definite. Letting $\mathbf{v} = \mathbf{L}^T\,\mathbf{u}_U$ and multiplying by $\mathbf{L}^T$, Eq. (2) can be rewritten as:

$$\frac{1}{g}\,\mathbf{L}^T\,\mathbf{k}_{UU}^{-1}\,\mathbf{L}\,\ddot{\mathbf{v}} + \mathbf{I}\,\mathbf{v} = \mathbf{0} \tag{3}$$

Equation (3) has solutions of the form $\mathbf{v} = \mathbf{V}\,e^{i\omega t}$, and in the classical form is:

$$[\mathbf{D} - \lambda\,\mathbf{I}]\,\mathbf{V} = \mathbf{0}$$

where

$$\mathbf{D} = \mathbf{L}^T\,\mathbf{k}_{UU}^{-1}\,\mathbf{L}$$

and

$$\lambda = g/\omega^2$$

When more than one solution is considered:

$$\mathbf{D}\mathbf{V} - \mathbf{V}\left[\diagdown\lambda\diagdown\right] = \mathbf{0}$$

The eigenvalues $\left[\diagdown\lambda\diagdown\right]$ and associated eigenvectors $\mathbf{V}$ of the dynamic matrix $\mathbf{D}$ have the order of the unrestrained terms of the weight matrix.

The normal modes in terms of the original degrees of freedom having weight are given by:

$$\mathbf{u}_{WN} = \mathbf{L}^{T^{-1}}\,\mathbf{V}$$

To expand these modes to the complete set of degrees of freedom, the original equilibrium equation with $\mathbf{f} = \mathbf{0}$ is restated as follows:

$$\mathbf{w}\,\mathbf{u}_N\left[\diagdown\frac{\omega^2}{g}\diagdown\right] - \mathbf{k}_{UU}\,\mathbf{u}_N = \mathbf{0}$$

From which it follows that:

$$\mathbf{k}_{UU}\,\mathbf{u}_N = \mathbf{w}_{UU}\,\mathbf{u}_{WN}\left[\diagdown\frac{\omega^2}{g}\diagdown\right]$$

The term $\left[\diagdown\omega^2/g\diagdown\right]$ only scales the vectors and will be omitted since normalization is arbitrary:

$$\mathbf{k}_{UU}\,\mathbf{u}_N = \mathbf{w}_{UU}\,\mathbf{u}_{WN} \tag{4}$$

Combining Eq. (1) and Eq. (4), the following expression is obtained:

$$[\mathbf{k}_{UU} \,\vdots\, \mathbf{k}_{UR}]\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix} = \left[\mathbf{f} + \mathbf{w}_{UU}\left[\ddot{\mathbf{u}}_U\frac{1}{g} + \mathbf{u}_{WN}\right]\right]$$

The reactions $\mathbf{f}_R$ can be calculated by multiplying the rows of $\mathbf{k}$ associated with prescribed motion by the displacement matrix $\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}$. In practice the reactions are calculated as a by-product of the solution of simultaneous equations.

The deflected shapes $\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}$ due to independent loading conditions, which include normal modes, can be used as transformation matrices. Deflections $\mathbf{u}$, which are linear combinations of these deflected shapes, can be defined as the product $\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}\mathbf{P}$. Elements of the vector $\mathbf{P}$ are the weighting factors or generalized displacements.

$$\mathbf{u} = \begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}\mathbf{P}$$

Differentiating both sides twice with respect to time gives:

$$\ddot{\mathbf{u}} = \begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}\ddot{\mathbf{P}}$$

Substituting into the equation of equilibrium gives:

$$\frac{1}{g}\,\mathbf{w}\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}\ddot{\mathbf{P}} + \mathbf{k}\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}\mathbf{P} = \mathbf{f}$$

Premultiplying by $\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}^T$ produces:

$$\frac{1}{g}\,\mathbf{W}\,\ddot{\mathbf{P}} + \mathbf{K}\,\mathbf{P} = \mathbf{F}$$

Where the generalized weight $\mathbf{W}$, generalized stiffness $\mathbf{K}$, and generalized force $\mathbf{F}$ are defined as follows:

$$\mathbf{W} = \begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}^T\mathbf{w}\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}^T\mathbf{k}\begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}$$

$$\mathbf{F} = \begin{bmatrix}\mathbf{u}_T\\ \cdots\\ \mathbf{u}_R\end{bmatrix}^T\mathbf{f}$$

Observe that

$$\mathbf{K} = \begin{bmatrix} \mathbf{u}_U \\ \hdashline \mathbf{u}_R \end{bmatrix}^T \begin{bmatrix} \mathbf{f}_U \\ \hdashline \mathbf{f}_R \end{bmatrix} = \begin{bmatrix} \mathbf{f}_U \\ \hdashline \mathbf{f}_R \end{bmatrix}^T \begin{bmatrix} \mathbf{u}_U \\ \hdashline \mathbf{u}_R \end{bmatrix}$$

and

$$K_{ij} = \begin{Bmatrix} \mathbf{u}_{Ui} \\ \hdashline \mathbf{u}_{Ri} \end{Bmatrix}^T \begin{Bmatrix} \mathbf{f}_{Uj} \\ \hdashline \mathbf{f}_{Rj} \end{Bmatrix} = \begin{Bmatrix} \mathbf{f}_{Ui} \\ \hdashline \mathbf{f}_{Ri} \end{Bmatrix}^T \begin{Bmatrix} \mathbf{u}_{Uj} \\ \hdashline \mathbf{u}_{Rj} \end{Bmatrix}$$

Let

$$K'_{ij} = \begin{Bmatrix} \mathbf{f}_{Ui} \\ \hdashline \mathbf{u}_{Ri} \end{Bmatrix}^T \begin{Bmatrix} \mathbf{u}_{Uj} \\ \hdashline \mathbf{f}_{Rj} \end{Bmatrix}$$

$$K'_{ij} = K_{ij}$$

if $\mathbf{f}_{Ui} = \mathbf{f}_{Uj} = \mathbf{0}$ as in modes defined by prescribed displacements, or $\mathbf{u}_{Ri} = \mathbf{u}_{Rj} = \mathbf{0}$ as in modes defined by forces.

If one mode is defined by displacement ($\mathbf{f}_{Ui} = \mathbf{0}$) and another by forces ($\mathbf{u}_{Rj} = \mathbf{0}$), the coupling element (K_{ij}) is zero. Therefore, in this mixed mode case

$$\mathbf{u}^T_{Ui}\, \mathbf{f}_{Uj} = -\, \mathbf{u}^T_{Ri}\, \mathbf{f}_{Rj}$$

$$\mathbf{f}^T_{Ui}\, \mathbf{u}_{Uj} = -\, \mathbf{f}^T_{Ri}\, \mathbf{u}_{Rj}$$

and

$$K'_{ij} = \mathbf{f}^T_{Ui}\, \mathbf{u}_{Uj} + \mathbf{u}^T_{Ri}\, \mathbf{f}_{Rj}$$

$$= -\, \mathbf{f}^T_{Ri}\, \mathbf{u}_{Rj} - \mathbf{u}^T_{Ui}\, \mathbf{f}_{Uj} = -\, K'_{ji}$$

Table 1. Matrix title and notation definitions

Title in SAMIS	Symbol in derivation	Definition
WTR 1	$\mathbf{w}$	Weights associated with degrees of freedom
ARC 1	$\ddot{\mathbf{u}}_R \frac{1}{g}$	Acceleration of restraints
KER 1	$\mathbf{k}_1$	Stiffness matrix for elements not modeled by SAMIS
KER 2	$\mathbf{k}_2$	Stiffness matrices for elements modeled by SAMIS
SCR 2	$\mathbf{S}$	Internal force coefficient matrices for elements modeled by SAMIS
N	—	Number of structural elements to be modeled by SAMIS
KTR 1	$\mathbf{k}$	Stiffness matrix of composite structure
P	—	N + 1
ATC 1	$\ddot{\mathbf{u}} \frac{1}{g}$	Total acceleration matrix
KUR 1	$\mathbf{U}$	Upper decomposition of restrained stiffness matrix with extra terms in restrained rows
WUR 1	$\mathbf{L}^T$	Upper decomposition of weight matrix
WLC 1	$\mathbf{L}$	Lower decomposition of weight matrix
TEC 1	$\mathbf{k}_U^{-1}\mathbf{L}$	—
DCC 1, DDC 1	$\mathbf{D}$	Dynamic matrix
VDC 1, VCC 1	$\mathbf{V}$	Eigenvectors of dynamic matrix
DWC 1	$\mathbf{u}_{WN}$	Normal mode shapes (degrees of freedom with weight only)
AUC 2	$\mathbf{u}_{WN} + \ddot{\mathbf{u}} \frac{1}{g}$	—
DFC 1	$\mathbf{w}\left[\mathbf{u}_{WN} + \ddot{\mathbf{u}} \frac{1}{g}\right]$	Inertia forces
FOC 1	$\mathbf{f}_U + \mathbf{u}_R$	Prescribed forces and displacements
FOC 2	$\mathbf{f}_U + \mathbf{u}_R + \mathbf{w}\left[\mathbf{u}_{WN} + \ddot{\mathbf{u}} \frac{1}{g}\right]$	—
DTC 1	$\begin{bmatrix} \mathbf{u}_T \\ \hdashline \mathbf{u}_R \end{bmatrix}$	Displacement matrix
DDC 1, GCC 1	$\begin{bmatrix} \mathbf{u}_T \\ \hdashline \mathbf{f}_R \end{bmatrix}$	Displacement and reaction matrix
FOR 2	$\begin{bmatrix} \mathbf{f}_U \\ \hdashline \mathbf{u}_R \end{bmatrix}^T$	Transposed prescribed force and displacement matrix
GCC 1, DDR 2	$\mathbf{K}'$	Matrix associated with generalized stiffness matrix
FOC 3	$\mathbf{w}\begin{bmatrix} \mathbf{u}_T \\ \hdashline \mathbf{u}_R \end{bmatrix}$	—
FOR 3	$\begin{bmatrix} \mathbf{u}_T \\ \hdashline \mathbf{u}_R \end{bmatrix}^T \mathbf{w}$	—
GCC 3, DDC 3	$\mathbf{W}$	Generalized weight
RCC 2, RDC 2	$\mathbf{R}$	Internal force matrix of first element

Therefore, $K_{ij} = (K'_{ij} + K'_{ji})/2$ in all cases including modes defined by a combination of forces and displacements.

The internal forces $\mathbf{R}$ in any member are linear combinations of the displacements of the ends of the member $\mathbf{u}_i$. A coefficient matrix $\mathbf{S}$ is generated for each member just as the elemental stiffness matrices are generated, and

$$\mathbf{R} = \mathbf{S}\,\mathbf{u}_i$$

The vectors $\mathbf{u}_i$ can be replaced by the vectors $\left\{\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right\}$ including all displacements, since the expansion of $\mathbf{S}$ by additional rows and columns of zeros will not change the elements of $\mathbf{R}$. Also, there may be more than one vector of displacement, and the following assembly occurs:

$$\mathbf{R} = \mathbf{S}\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right]$$

A listing of the required SAMIS pseudoinstructions (see Fig. 3) and a specialized subset omitting normal modes (see Fig. 4) are provided later in this report. Displacements or reactions associated with each degree of freedom and member internal forces are calculated and printed in all cases. Generalized weight and generalized stiffness are calculated and printed only if normal modes are calculated. Table 1 lists the alphanumeric title of each matrix (used in the pseudoinstructions), the symbol used in the derivation, and a brief definition. The operation performed by each pseudoinstruction is stated (in terms of matrix operation where possible) in Table 2. The preparation of data and arrangement of other required inputs are explained in Section III.

Table 2. Pseudoinstruction functions

Instruction	Function
0.0	Data check
1.0	Basic data $\mathbf{w}\ \ddot{\mathbf{u}}\ \mathbf{k}_1$ input and printed
2.0	$\mathbf{k}_i$, $\mathbf{S}_i$, $i = 2, N+1$ generated from geometric data and material properties and data printed
3.0	$\mathbf{k} = \Sigma\,\mathbf{k}_i, i = 1, N+1$
4.0,4.1	Solves for $\mathbf{U}$ and $\dfrac{\ddot{\mathbf{u}}_U}{\ddot{\mathbf{u}}_R}$ in $\left[\begin{array}{c:c}\mathbf{k}_{UU} & \mathbf{k}_{UR}\\ \hdashline \mathbf{k}_{RU} & \mathbf{k}_{RR}\end{array}\right]\left[\begin{array}{c}\ddot{\mathbf{u}}_U\\ \hdashline \ddot{\mathbf{u}}_R\end{array}\right] = \left[\begin{array}{c}\mathbf{0}\\ \hdashline \mathbf{f}_R\end{array}\right]$ where $\mathbf{U}^T_{UU}\mathbf{U}_{UU} = \mathbf{k}_{UU}$
5.0,5.3	Instructions 5.1 and 5.2 implemented only if $[\mathbf{k}_{UU}]$ not positive definite, in which case offending rows and columns restrained in 4.0
5.2	Solves for $\left[\begin{array}{c}\mathbf{u}_U\\ \hdashline \mathbf{I}\end{array}\right]$ in $\left[\begin{array}{c:c}\mathbf{k}_{UU} & \mathbf{k}_{UR}\\ \hdashline \mathbf{k}_{RU} & \mathbf{k}_{RR}\end{array}\right]\left[\begin{array}{c}\mathbf{u}_U\\ \hdashline \mathbf{I}\end{array}\right] = \left[\begin{array}{c}\mathbf{0}\\ \hdashline \mathbf{f}_R\end{array}\right]$ with added restraint in $\mathbf{k}$
5.3	Outputs $\left[\begin{array}{c}\mathbf{u}_U\\ \hdashline \mathbf{I}\end{array}\right]$ as diagnostic and terminates problem
6.0,6.1	Solves for $\mathbf{L}^T$ where $\mathbf{L}\,\mathbf{L}^T = \mathbf{w}$
7.0	Transposes $\mathbf{L}^T$
8.0	Solves for $\left[\begin{array}{c}\mathbf{X}\\ \hdashline \mathbf{0}\end{array}\right]$ in $\left[\begin{array}{c:c}\mathbf{k}_{UU} & \mathbf{k}_{UR}\\ \hdashline \mathbf{k}_{RU} & \mathbf{k}_{RR}\end{array}\right]\left[\begin{array}{c}\mathbf{X}\\ \hdashline \mathbf{0}\end{array}\right] = \left[\begin{array}{c}\mathbf{L}\\ \hdashline \mathbf{f}_R\end{array}\right]$
9.0	Multiplies $\mathbf{L}^T\,\mathbf{X} = \mathbf{D}$
10.0	Format change
11.0	Generates six lowest-frequency eigenvalues of $\mathbf{D}$ and associated eigenvectors $\mathbf{V}$; frequencies printed
12.0	Format change
13.0	Solves for $\mathbf{u}_{WN}$ in $\mathbf{L}^T\,\mathbf{u}_{WN} = \mathbf{V}$
14.0	Adds $\left[\begin{array}{c}\ddot{\mathbf{u}}_U\\ \hdashline \ddot{\mathbf{u}}_R\end{array}\right]\dfrac{1}{g} + \mathbf{u}_{WN}$
15.0	Multiplies $\mathbf{w}\left[\left[\begin{array}{c}\ddot{\mathbf{u}}_U\\ \hdashline \ddot{\mathbf{u}}_R\end{array}\right]\dfrac{1}{g} + \mathbf{u}_{WN}\right]$
16.0	Basic data $\left[\begin{array}{c}\mathbf{f}_U\\ \hdashline \mathbf{u}_R\end{array}\right]$ input and printed
17.0	Adds $\left[\begin{array}{c}\mathbf{f}_U\\ \hdashline \mathbf{u}_R\end{array}\right] + \mathbf{w}\left[\left[\begin{array}{c}\ddot{\mathbf{u}}_U\\ \hdashline \ddot{\mathbf{u}}_R\end{array}\right]\dfrac{1}{g} + \mathbf{u}_{WN}\right] = \left[\begin{array}{c}\mathbf{X}\\ \hdashline \mathbf{u}_R\end{array}\right]$
18.0	Solves for $\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right]$ and $\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{f}_R\end{array}\right]$ in $\left[\begin{array}{c:c}\mathbf{k}_{UU} & \mathbf{k}_{UR}\\ \hdashline \mathbf{k}_{RU} & \mathbf{k}_{RR}\end{array}\right]\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right] = \left[\begin{array}{c}\mathbf{X}\\ \hdashline \mathbf{f}_R\end{array}\right]$
19.0	Data transfer
20.0	Transposes $\left[\begin{array}{c}\mathbf{X}\\ \hdashline \mathbf{u}_R\end{array}\right]$
21.0	Multiplies $\left[\begin{array}{c}\mathbf{X}\\ \hdashline \mathbf{u}_R\end{array}\right]^T\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{f}_R\end{array}\right] = \mathbf{K}'$
22.0	Multiplies $\mathbf{w}\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right]$
23.0	Transposes $\mathbf{w}\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right]$
24.0	Multiplies $\left[\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right]^T\mathbf{w}\right]\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right] = \mathbf{W}$
25.0	Format change
26.0	Outputs $\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{f}_R\end{array}\right]^T$, $\mathbf{K}'$, $\mathbf{W}$
27.0	Data transfer
28.0	Multiplies $\mathbf{S}_i\left[\begin{array}{c}\mathbf{u}_T\\ \hdashline \mathbf{u}_R\end{array}\right] = \mathbf{R}_i$ for $i = 2, N+1$
29.0	Format change
30.0	Outputs $\mathbf{R}_i$ for $i = 2, N+1$
31.0	Computations complete

C. Structural Idealization

The derivation of the line element stiffness matrix used is given in Ref. 2.

The data formats for structures composed of line elements will be given (see Fig. 6) for three special cases. The first case is for three-dimensional structures with members having equal bending stiffness about any axis normal to the member and with the centroid of the member cross section and shear center on a straight line between the end points. The second case is for planar structures in the X-Y plane and loaded in this plane for which only the first, second, and sixth degrees of freedom at each gridpoint are generated. The third case is for planar structures in the X-Y plane and loaded out of the plane for which only the third, fourth, and fifth degrees of freedom at each gridpoint are generated.

If a member section property is given a zero value, the associated contribution to the stiffness matrix is zero. If the contribution to the stiffness matrix of all members at a gridpoint is zero along a coordinate axis, the degree of freedom at that gridpoint along that axis is not generated and the structure acts as though that degree of freedom were restrained. If the contribution to the stiffness matrix of all members at a gridpoint is zero in a direction not along a coordinate axis, and if a vector in that direction has no component in the direction of a restraint at that gridpoint, the analytic model has a singular stiffness matrix and the associated structure is unstable.

III. Use of SAMIS for Structural Analysis Problems

Sections III and IV are intended for use independent of the remainder of the report so that the information can be used without detailed knowledge of SAMIS. The analyst familiar with SAMIS can make useful extensions by taking advantage of other options within SAMIS, some of which are enumerated in Section V. Conservative time estimates for an IBM DCS 7044/7094 computer can be obtained from Fig. 1.

A. Limitations

Structural analysis problems can be analyzed using SAMIS within the following limitations.

(1) The stiffness and weight matrices as restrained, considering only rows and columns with entries, must be positive definite.

(2) The number of unrestrained degrees of freedom with weight must be ≤ 140 if normal modes are required.

(3) The product of the number of independent loading conditions (including six normal modes if calculated) and the number of degrees of freedom both restrained and unrestrained must be $\leq 10{,}000$.

(4) The number of independent loading conditions (including six normal modes if calculated) must be ≤ 140.

(5) Row and column codes must be $\leq 40{,}959$ (gridpoints $\leq 4{,}095$).

(6) There must be no weight associated with restrained degrees of freedom.

(7) The number of structural elements (N) must be ≤ 996.

(8) The number of nonzero elements in WTR 1 must be $\leq 10{,}000$.

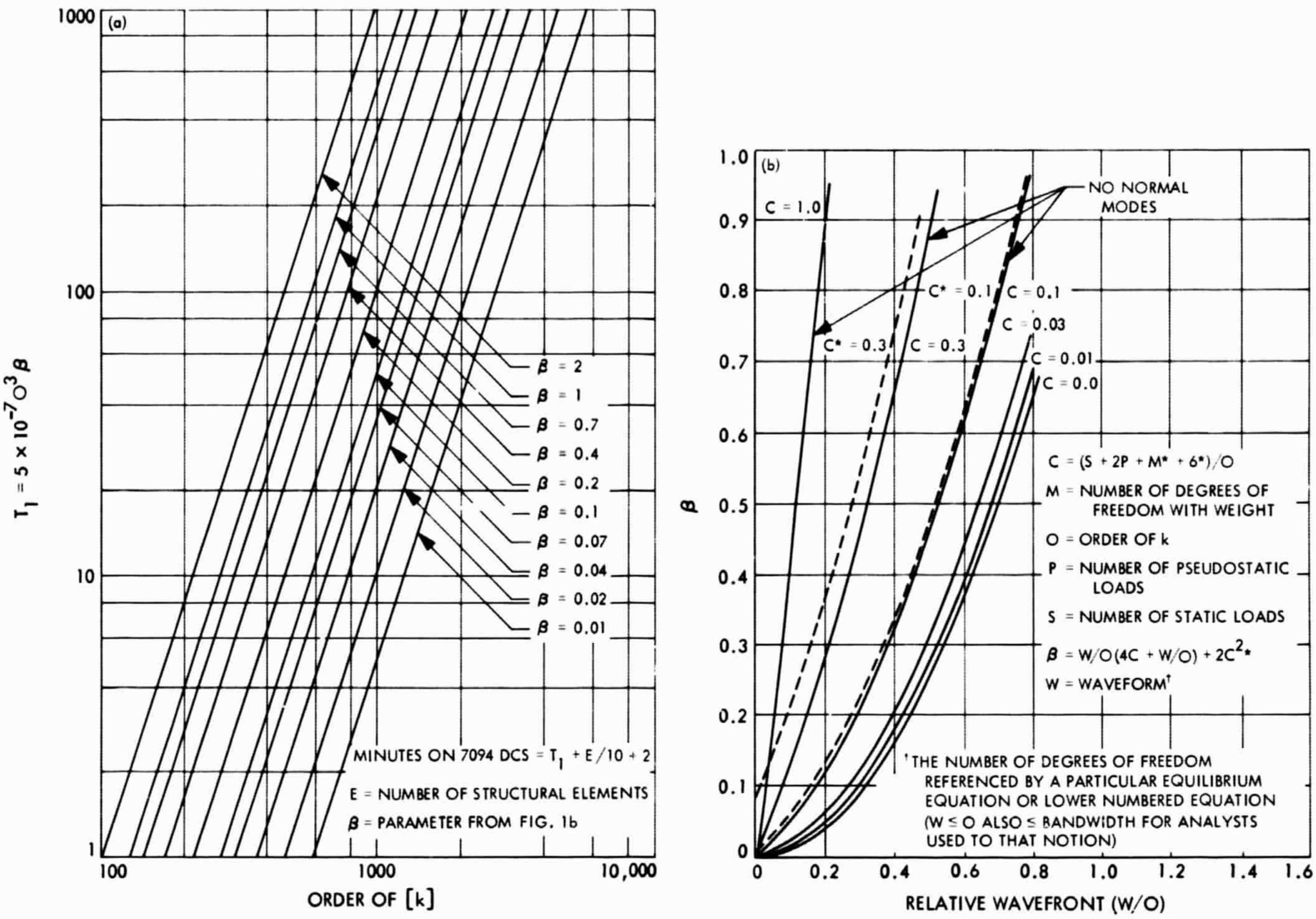
(9) The product of the number of independent loading conditions and unrestrained degrees of freedom with weight plus elements of FOC 1 must be $\leq 10{,}000$.

B. Input Data

The input can be divided into six types, which must be punched and located within the input deck as shown in Fig. 2. The six types of input are:

(1) An initiating deck. This deck is required by SAMIS and can be treated as the program. A description of its preparation is given in Ref. 1 and in the material distributed with the program.

(2) Control cards. The control cards, which direct computations, are listed in Fig. 3 for the most general case of static and pseudostatic loading and normal modes in a single analysis. For efficiency, if only static and pseudostatic loading are required, use the control cards of Fig. 4. Material control cards and matrix control cards are shown in Fig. 5. The number of structural elements (N) ($P = N + 1$ in one case), the number of cards used ($N1$, $N2$, $N3$, $N4$, and $N5$) for type 3 and type 6 inputs, and material properties (E and G) must be given a numerical value on 12 of these control cards.



$$\text{LINES OF OUTPUT} = \frac{L^{2*}}{4} + \frac{OL}{8} + E\,(L + 7) + 1000$$

WHERE L = S + P + 6* (THE TERMS WITH * INCLUDED ONLY IF NORMAL MODES ARE CALCULATED)

Fig. 1. Running time and line count estimates

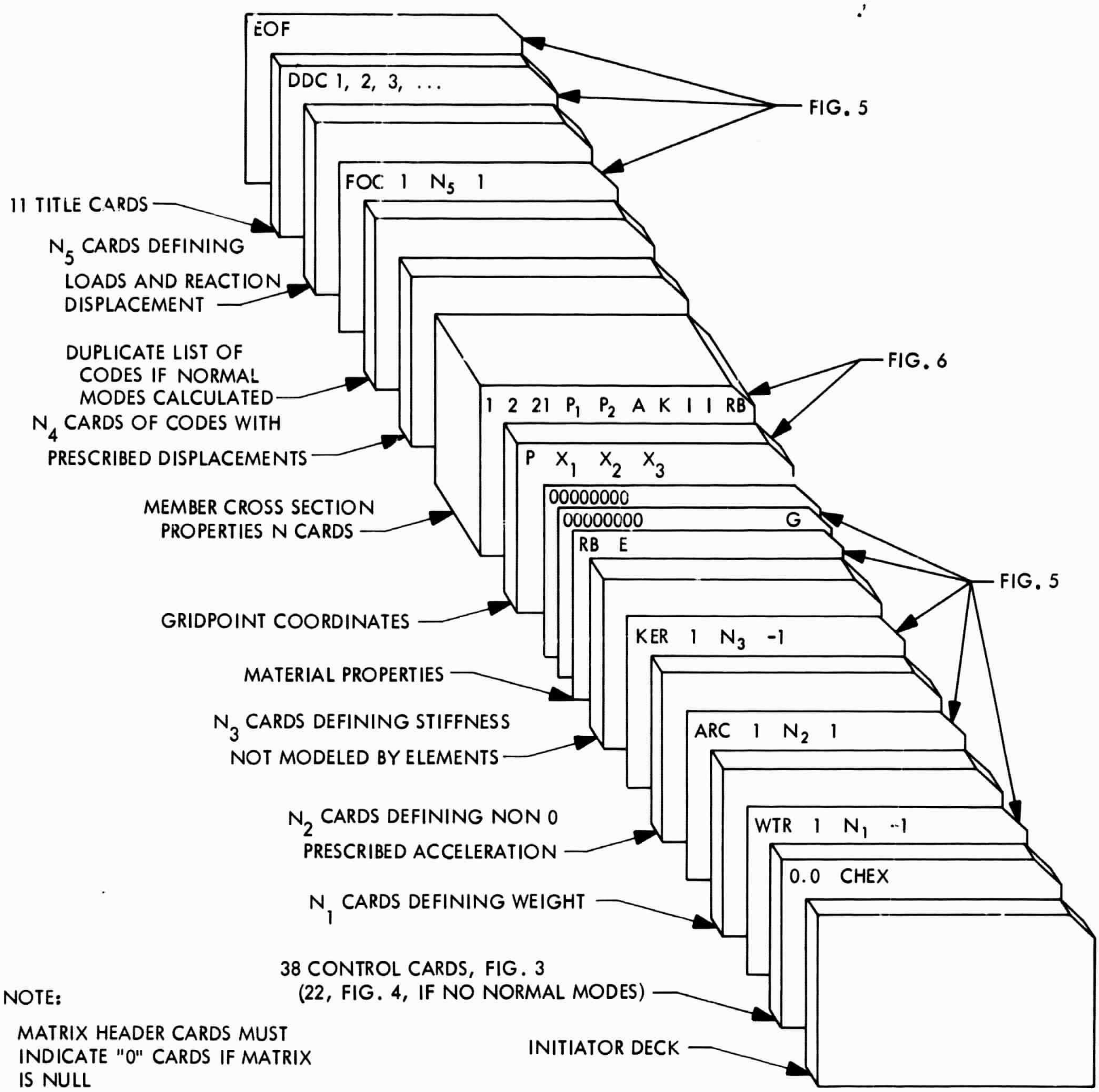


Fig. 2. Input deck arrangement

FORTRAN CODING FORM

	Punching Instructions		Page 1 of 2
Program STANDARD SAMIS	Graphic	Card Form #	Identification
Programmer BAMFORD Date 1/5/67	Punch		73 – 80

C FOR COMMENT

Statement Number (1–5)	Cont. (6)	7–10	11–16	17–24	25–30	31–39	40–48	49–54	55–64	65–71	72
0.0							CHEX				1
1.0			9001	WTR 1	9002	ARC 1	READ	9003	KER 1	-1	
2.0			9004	KER 2	10001	SCR 2	BILD			-N05	
3.0			9003	KER 1			ADDS	12001	KTR 1	P	
4.0			12001	KTR 1	9002	ARC 1	CHØL	12002	ATC 1	510	N4
4.1					11001	KUR 1	CØNT				
5.0							ERRS				
5.1			11001	KUR 1			CHØL	9001	RBC999	920	
5.2			9001	RBC999			INKS				
5.3							STØP				
6.0			9001	WTR 1			CHØL				N4
6.1					11002	WUR 1	CØNT				
7.0			11002	WUR 1			FLIP	12003	WLC 1		
8.0			11001	KUR 1	12003	WLC 1	CHØL	9002	TEC 1	900	
9.0			11002	WUR 1	9002	TEC 1	MULT	12003	DCC 1		
10.0			12003	DCC 1			DECØ	11003	DDC 1		
11.0			11003	DDC 1	11004	VDC 1	RØØT				-6
12.0			11004	VDC 1			CØDE	9002	VCC 1		
13.0			11002	WUR 1	9002	VCC 1	CHØL	12003	DWC 1	890	
14.0			12002	ATC 1	12003	DWC 1	ADDS	11005	AUC 2		
15.0			9001	WTR 1	11005	AUC 2	MULT	9002	DFC 1		

Fig. 3. Facsimile of operational control cards (including normal modes)

FORTRAN CODING FORM

Program	STANDARD SAMIS		Punching Instructions		Page 2 of 2
Programmer	BAMFORD	Date 1/5/67	Graphic / Punch	Card Form #	Identification 73–80

C FOR COMMENT

Statement Number (1–5)	Cont. (6)	FORTRAN Statement (7–18)	(19–24)	(25–30)	(31–36)	(37–48)	(49–54)	(55–66)	(67–72)
16.0			FØC 1			READ			-1
17.0			FØC 1	9002	DFC 1	ADDS	12001	FØC 2	
18.0		11001	KUR 1	12001	FØC 2	CHØL	9002	DTC 1	910
18.1		12002	DDC 1			CØNT			
19.0		12002	DDC 1			CØLS	11006	GCC 1	
20.0		12001	FØC 2			FLIP		FØR 2	
21.0			FØR 2	12002	DDC 1	MULT	11007	GCC 2	
22.0		9001	WTR 1	9002	DTC 1	MULT	12003	FØC 3	
23.0		12003	FØC 3			FLIP		FØR 3	
24.0			FØR 3	9002	DTC 1	MULT	11008	GCC 3	
25.0		11006	GCC 1			DECØ	12002	DDC 1	3
26.0		12002	DDC 1			INKS			305
27.0		9002	DTC 1			FILL			
28.0		10001	SCR 2		DTC 1	MULT	11003	RCC 2	N
29.0		11003	RCC 2			DECØ	12001	RDC 2	N
30.0		12001	RDC 2			INKS			N06
31.0						HALT			

NOTE: Replace N by the number of structural elements and P by N + 1 (I6)

Replace N_4 by the number of cards of codes associated with prescribed displacements (I2)

Fig. 3 (contd)

(3) Codes and elements of four matrices required to define the model and its loading:

Title	*Description*
WTR 1	The nonzero elements of the weight matrix. This matrix, considering only rows and columns with entries, must be symmetric and positive definite and of order 130 or less after restraint; but need not be diagonal or of the same order as $[k]$. No weight is to be associated with restrained degrees of freedom.
ARC 1	Nonzero accelerations associated with restraints (type 6 input defines restrained degrees of freedom). The units of these values are the acceleration due to gravity (g) for degrees of freedom 1, 2, and 3 at any joint, and g/in. for degrees of freedom 4, 5, and 6 [3(2I6, E12.0)].
KER 1	The nonzero elements of the stiffness matrix that are not contributed by members listed in element data. This matrix must be positive semidefinite and only elements with column codes ≥ row codes (upper half including diagonal of matrix) are required and used [3(2I6, E12.0].
FOC 1	The nonzero boundary conditions (applied loads or displacements). Prescribed displacements are associated with restrained degrees of freedom (type 6 input). At any joint displacements in degrees of free-

FORTRAN CODING FORM

Program		Punching Instructions		Page of
Programmer	Date	Graphic / Punch	Card Form #	Identification 73–80

C FOR COMMENT

STATEMENT NUMBER (1–5)	FORTRAN STATEMENT (10–15)	(16–24)	(25–30)	(31–39)	(40–48)	(49–54)	(55–63)	(64–72)
0.0					CHEX			1
1.0	9001	WTR 1	9002	ARC 1	READ	9003	KER 1	-1
2.0	9004	KER 2	10001	SCR 2	BILD			-NOS
3.0	9003	KER 1			ADDS	12001	KTR 1	E
4.0	12001	KTR 1	9002	ARC 1	CHØL	12002	ATC 1	510 N
4.1			11001	KUR 1	CØNT			
5.0					ERRS			
5.1	11001	KUR 1			CHØL	9001	RAC999	920
5.2	9001	RAC999			INKS			
5.3					STØP			
15.0	9001	WTR 1	12002	ATC 1	MULT	9002	DFC 1	
16.0		FØC 1			READ			-1
17.0		FØC 1	9002	DFC 1	ADDS	12001	FØC 2	
18.0	11001	KUR 1	12001	FØC 2	CHØL	9002	DTC 1	910
18.1	12002	DDC 1			CØNT			
25.0	12002	DDC 1			DECØ	11002	DDC 1	1
26.0	11002	DDC 1			INKS			105
27.0	9002	DTC 1			FILL			
28.0	10001	SCR 2		DTC 1	MULT	11003	ROC 2	M
29.0	11003	ROC 2			DECØ	12001	RDC 2	N
30.0	12001	RDC 2			INKS			NOE
31.0					HALT			

Fig. 4. Facsimile of operational control cards (no normal modes)

dom 1, 2, 3 are in in. and in degrees of freedom 4, 5, 6 are in rad. Loads are associated with unrestrained degrees of freedom. At any joint loads in degrees of freedom 1, 2, 3 are in lb and in degrees of freedom 4, 5, 6 are in in.-lb. [3(2I6, *E*12.0)].

The elements of the required four matrices are read in the format [3(2I6, *E*12.0)] up to three elements per card with the row and column codes of each element preceding the element. The order of elements is important. In WTR 1 and KER 1, all elements of a row precede elements of higher-numbered rows. Within a row, the elements are ordered by increasing column number. In ARC 1 and FOC 1, all elements of a column precede elements of higher-numbered columns. Within the column, the elements are ordered by increasing row number. The first digits of the row numbers (and column numbers of WTR 1 and KER 1) correspond to gridpoints, and the last digit corresponds to the degrees of freedom (from 1 to 6) at the gridpoint. Degrees of freedom 4, 5, and 6 are rotations about axes 1, 2, and 3. The columns of ARC 1 and FOC 1 correspond to the loading condition (an arbitrary number from 7 to 40,959). The FOC 1 data of the example problem are expanded into matrix format (see Fig. 9) for comparison with the input data.

(4) Tabulated gridpoint coordinates, as shown in Fig. 6.

(5) Member cross-section geometric properties, as shown in Fig. 6. These properties are for three types of structures, each composed of line elements. The first group is three-dimensional structures

FORTRAN CODING FORM

Program		Punching Instructions: Graphic / Punch	Card Form #	Page of / Identification 73–80
Programmer	Date			

C FOR COMMENT — STATEMENT NUMBER 1–5 | Cont. 6 | FORTRAN STATEMENT 7–72

```
    RB                          E
00000000                                                        G
00000000
WTR  1     N1               -1
ARC  1     N2                1
KER  1     N3               -1
FØC  1     N5                1
 DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GEN WEIGHT MATRICES
 RØW AND CØLUMN CØRRESPØND TØ MØDE, LØAD, ØR REACTIØN DISPLACEMENT ØR ACC
 EXCEPT DDC 1 RØW LEADING DIGITS CØRRESPØND TØ GRIDPØINT AND FINAL DIGIT
 TØ DEGREE ØF FREEDØM AT THE GRIDPØINT
 REACTIØN NØT DISPLACEMENT AT RESTRAINT
 INTERNAL FØRCE FØR ELEMENT X ( RDC X )
 RØW LEADING DIGITS CØRRESPØND TØ GRIDPØINT, FINAL DIGIT TØ
                           1 AXIAL FØRCE          4 TØRQUE
                           8 SHEAR ALØNG X2       5 MØMENT ABØUT X2
                           7 SHEAR ALØNG X3       9 MØMENT ABØUT X3
 CØL CØRRESPØNDS TØ MØDE, LØAD ØR REACT. DISP., ØR ACC.
```

Replace "E" and "G" by the numerical value of Young's modules and the modulus of rigidity (E8.0)

Replace N_1, N_2, N_3, N_5 by the number of cards of data in each matrix (I6)

Fig. 5. Facsimile of material property, matrix control, and title cards

with members having equal bending stiffness about any axis normal to the member and with the centroid of their cross sections and shear centers along straight lines between their end points. The second group is planar structures in the *X-Y* plane loaded in plane for which only the first, second, and sixth degrees of freedom at each gridpoint are generated. The third group is planar structures in the *X-Y* plane and loaded out of the plane for which only the third, fourth, and fifth degrees of freedom at each gridpoint are generated. If a member section property is given a zero value, the associated contribution to the stiffness matrix is zero. If the contribution to the stiffness matrix of all members at a gridpoint is zero along a coordinate axis, that degree of freedom is not generated and the structure acts as though that degree of freedom were restrained. If the contribution to the stiffness matrix of all members at a gridpoint is zero in a direction not along a coordinate axis and no component of a vector in that direction is along a restraint at that gridpoint, the analytic model has a singular stiffness matrix and the associated structure is unstable.

(6) Codes in ascending order that correspond to prescribed displacements. No implication is made that nonzero values will be prescribed, but if the displacement or acceleration is to be prescribed, the code must be included and the corresponding degree of freedom will be restrained (12 *I*6). Two lists are supplied to be used when normal modes are calculated.

FORTRAN CODING FORM

Program		Punching Instructions: Graphic / Punch	Card Form #	Page of / Identification 73–80
Programmer	Date			

C FOR COMMENT — STATEMENT NUMBER — FORTRAN STATEMENT

gridpoint coordinates X_1, X_2, X_3 (one card for each gridpoint P in ascending order: 3X, I4, 3E7)

P X_1 X_2 X_3

line element with equal stiffness about all normals to element

1 i 21 P_1 P_2 A K I_1 I RB

line element in x_1x_2 plane and loaded in plane

1 i 21 P_1 P_2 A I_3 RB

line element in x_1x_2 plane and loaded out of plane

1 i 21 P_1 P_2 K I_2 RB

i = element number (start at 2 and increment by 1) (I3)

P_1 = gridpoint at end of member (3 x I4)

P_2 = gridpoint at opposite end of member (3 x I4)

A = member area (in.^{2}) (E7.0)

K = effective polar moment of inertia (in.^{4}) (E7.0)

I = moment of inertia about all normals to GP_1, GP_2 (in.^{4}) (E7.0)

I_3 = moment of inertia about normal to plane (in.^{4}) (E7.0)

I_2 = moment of inertia about in plane normal to GP_1, GP_2 (in.^{4}) (E7.0)

Fig. 6. Gridpoint coordinates and element properties

C. Output Data

Echoes of input control cards, matrices, and member cross-section geometric properties are printed. The coordinates of the gridpoints defining each member are also printed with the member properties.

Displacements of each degree of freedom are printed, except reactions are printed instead of restraint displacements. Eigenvalues for the six lowest frequency normal modes and generalized weight matrix and a matrix ($\mathbf{K}'$) associated with the stiffness matrix ($\mathbf{K}$) are printed if the control cards of Fig. 3 are used. This relationship between $\mathbf{K}'$ and $\mathbf{K}$ is expressed by the following equation:

$$K_{ij} = (K'_{ij} + K'_{ji})/2$$

If two associated modes are both defined by forces or both defined by displacements, $K'_{ij} = K_{ij}$. The element $K_{ij} = 0$, if one of the associated modes is defined by displacement and the others by forces. The generalized weight associated with normal modes is normalized to g^2/ω^4 and the generalized stiffness to g/ω^2. The natural frequencies of these modes ($\omega/2\pi$) are given in the last column of the printout entitled "Roots of Matrix DDC 1," which follows the element data in the sample problem output. Internal force matrices for each structural element are printed. The coordinate system for internal forces has the X_1 axis along a vector from the first gridpoint to the second gridpoint of the element (as defined by the element data input). The X_3 axis is along the cross product of this vector and a second vector from the first gridpoint to the origin. The X_2 axis is along the cross product of the vectors along the X_3

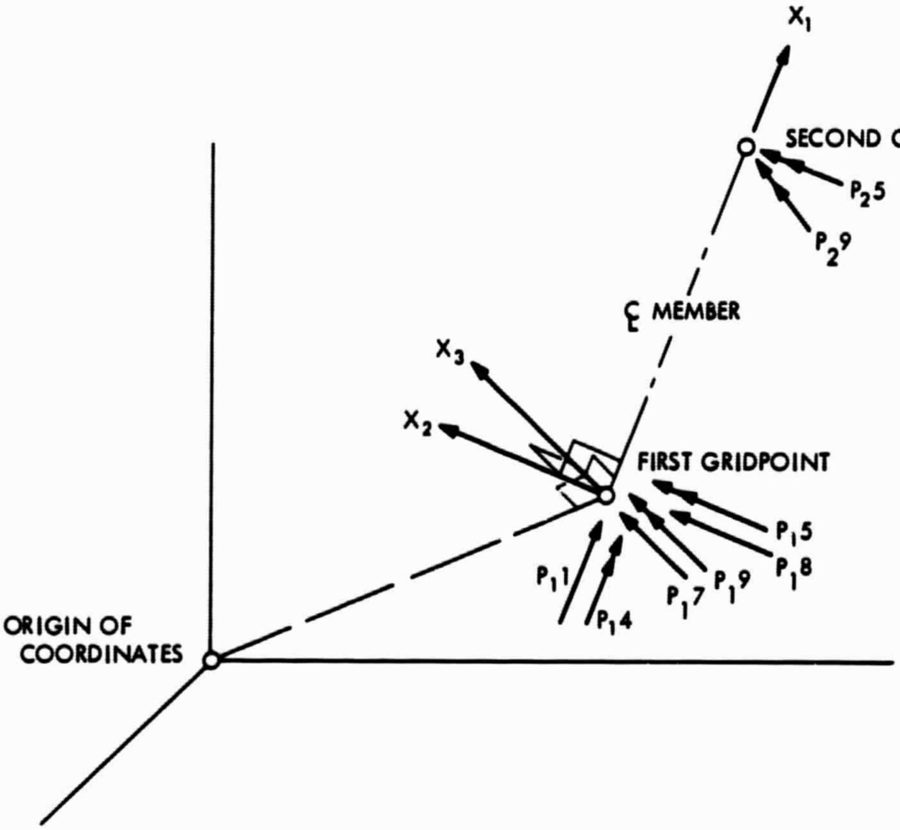


Fig. 7. Coordinate system for member loads

and X_1 axis, as shown in Fig. 7. (Times and data locations are printed between SAMIS operations; refer to Ref. 3 for interpretation of this material, which is beyond the scope of this report.) If the two points are colinear with the origin, the point (1, 2, 0) is used instead of the origin. If (1, 2, 0) is colinear, the point (4.3333, 3.42826, 0) is used. The leading digits of the internal force row codes are the gridpoint number, and the final digits are:

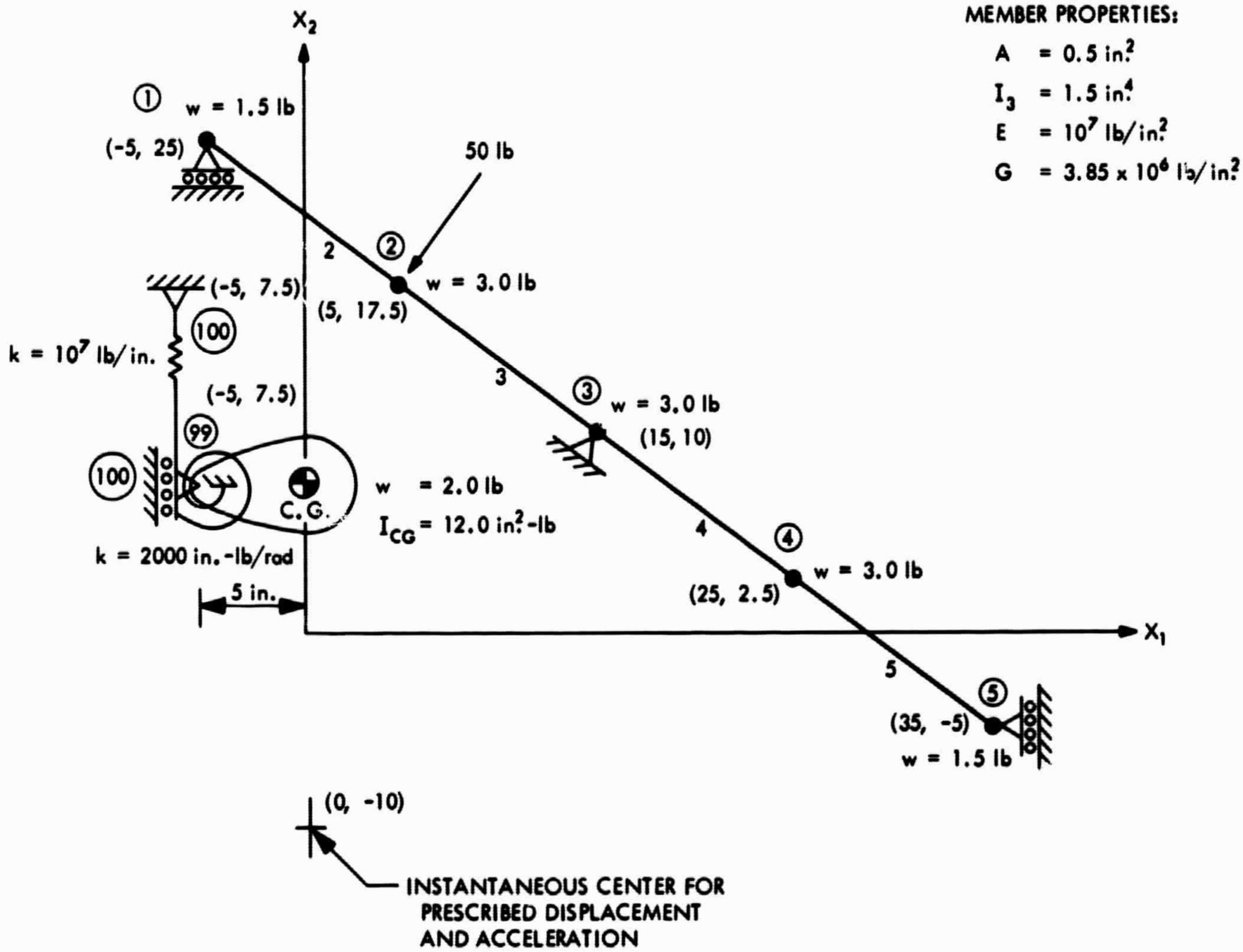
Component	*Description*
1	Axial force
8	Shear along X_2 axis
7	Shear along X_3 axis
4	Twisting moment
5	Moment about X_2
9	Moment about X_3

IV. Sample Problem

The problem chosen (Fig. 8 gives geometry and definition of loadings and Fig. 9 identifies the elements of matrix FOC 1) utilizes the most general set of pseudo-instructions (Fig. 3) and exercises all loading options. In addition to the six normal modes and a concentrated load, two distinct loading conditions are defined by reaction displacement, one of which is compatible with rigid body motion (rigid body mode). The same rigid body motion is used to define an acceleration field about an arbitrary instantaneous center and the associated inertial loading.

A last example shows that, by using the same column code for different types of loads, composite load conditions are generated. The example is the sum of the static, pseudostatic, and foundation settlement loads. The elements chosen are in the X-Y plane and loaded only in that plane.

In addition to a beam formed with four members and six unrestrained translational weights, an independent rotational weight is supported through a rotational spring to illustrate the use of nonmember stiffnesses and the necessity of including even the restrained degrees of freedom of the stiffness matrix when rigid body modes or pseudostatic loads are generated. The effect of a rotational acceleration on such weights is also illustrated.



LOADING COL 101	0.01-rad RIGID BODY ROTATION ABOUT (0, -10)
LOADING COL 102	-1 in. PRESCRIBED DISPLACEMENT X_2 DEGREE OF FREEDOM POINT 1 (FOUNDATION SETTLEMENT)
LOADING COL 201	50-lb LOAD AT GRIDPOINT 2 NORMAL TO BEAM
LOADING COL 202	0.01 g/in. PSEUDOSTATIC LOADING ABOUT (0, -10); NOTICE ONLY ROTARY ACCELERATION AFFECTS GRIDPOINT 99
LOADING COL 301	COMPOSITE OF LOADINGS 102, 201, AND 202

Fig. 8. Sample problem geometry and loading

FOC 1 =

ROW \ COL	101	102	201	301
12	-0.05	-1.0	0	-1.0
21	0	0	-30	-30
22	0	0	-40	-40
31	-0.20	0	0	0
32	0.15	0	0	0
51	-0.05	0	0	0
1002	-0.05	0	0	0
1006	0.01	0	0	0

Fig. 9. Expanded static load matrix for sample problem

The eigenvalue problem is of order 8 due to the eight unrestrained degrees of freedom with weight even though there are 13 unrestrained degrees of freedom. The weight at joint 99 illustrates the use of weights eccentric to the gridpoint through off-diagonal elements of the weight matrix. Since weight at restrained degrees of freedom is not allowed, a stiff spring to a restraint at gridpoint 100 is used at joint 99 in the second degree of freedom.

The computer printout of the input deck (exclusive of the starter deck) and the output for the sample problem are presented in the appendix of this report.

V. Extensions

Extensions included in SAMIS that are not described in detail in this report are enumerated in the following list; for more information the analyst is referred to Refs. 1 and 3. Only the last three extensions require modification of the pseudoinstructions:

(1) Restart with a minimum of repeated calculations. Pseudoinstructions were written with the intention of saving tapes 10 and 11.

(2) Unequal moments of inertia, shear stiffness, and other structural elements.

(3) Eccentricities (substitute gridpoints).

(4) Discontinuities (such as hinges).

(5) Local coordinates for:

- (a) Nonisotropic material property reference.
- (b) Gridpoint location reference.
- (c) Displacement reference.
- (d) Stress resultant reference.

(6) Multiple materials including nonisotropic materials.

(7) Punched output in addition to printed.

(8) Modification of pseudoinstructions.

(9) Self-generated elemental mass and the following loading matrices:

- (a) Thermal.
- (b) Pressure and line loads.
- (c) Gravity.

References

1. Melosh, R. J., Diether, P. A., and Brennan, M., *Structural Analysis and Matrix Interpretive System (SAMIS) Program Report,* Technical Memorandum 33-307, Rev. 1. Jet Propulsion Laboratory, Pasadena, Calif., Dec. 15, 1966.
2. Melosh, R. J., and Christiansen, H. N., *Structural Analysis and Matrix Interpretive System (SAMIS) Technical Report,* Technical Memorandum 33-311. Jet Propulsion Laboratory, Pasadena, Calif., Nov. 1, 1966.
3. Lang, T. E., *Structural Analysis and Matrix Interpretive System (SAMIS) User Report,* Technical Memorandum 33-305. Jet Propulsion Laboratory, Pasadena, Calif., Mar. 1, 1967.

Appendix

Inputs and Outputs of Sample Problem

Input

```
  0.0                                              CHEX                           1
  1.0     9001  WTR  1      9002  ARC  1           READ     9003  KER  1         -1
  2.0     9004  KER  2     10001  SCR  2           BILD                        -405
  3.0     9003  KER  1                             ADDS    12001  KTR  1          5
  4.0    12001  KTR  1      9002  ARC  1           CHOL    12002  ATC  1      510 1
  4.1                      11001  KUR  1           CONT
  5.0                                              ERRS
  5.1    11001  KUR  1                             CHOL     9001  RBC999      92000
  5.2     9001  RBC999                             INKS
  5.3                                              STOP
  6.0     9001  WTR  1                             CHOL                           1
  6.1                      11002  WUR  1           CONT
  7.0    11002  WUR  1                             FLIP    12003  WLC  1
  8.0    11001  KUR  1     12003  WLC  1           CHOL     9002  TEC  1      900
  9.0    11002  WUR  1      9002  TEC  1           MULT    12003  DCC  1
 10.0    12003  DCC  1                             DECO    11003  DDC  1
 11.0    11003  DDC  1     11004  VDC  1           ROOT                          -6
 12.0    11004  VDC  1                             CODE     9002  VCC  1
 13.0    11002  WUR  1      9002  VCC  1           CHOL    12003  DWC  1      89000
 14.0    12002  ATC  1     12003  DWC  1           ADDS    11005  AUC  2
 15.0     9001  WTR  1     11005  AUC  2           MULT     9002  DFC  1
 16.0           FOC  1                             READ                          -1
 17.0           FOC  1      9002  DFC  1           ADDS    12001  FOC  2
 18.0    11001  KUR  1     12001  FOC  2           CHOL     9002  DTC  1      91000
 18.1    12002  DDC  1                             CONT
 19.0    12002  DDC  1                             COLS    11006  GCC  1
 20.0    12001  FOC  2                             FLIP           FOR  2
 21.0           FOR  2     12002  DDC  1           MULT    11007  GCC  2
 22.0     9001  WTR  1      9002  DTC  1           MULT    12003  FOC  3
 23.0    12003  FOC  3                             FLIP           FOR  3
 24.0           FOR  3      9002  DTC  1           MULT    11008  GCC  3
 25.0    11006  GCC  1                             DECO    12002  DDC  1      3
 26.0    12002  DDC  1                             INKS                         305
 27.0     9002  DTC  1                             FILL
 28.0    10001  SCR  2            DTC  1           MULT    11009  RCC  2      4
 29.0    11009  RCC  2                             DECO    10001  RDC  2      4
 30.0    10001  RDC  2                             INKS                         406
 31.0                                              HALT
WTR  1     4                  -1
    11    11  1.5             21    21  3.0               22    22  3.0
    41    41  3.0             42    42  3.0               52    52  1.5
   992   992  2.0            992   996 10.0              996   992 10.0
   996   996 62.0
ARC  1     4                   1
    12   202-.05              31   202-0.2                32   202 0.15
    51   202-.05            1002   202-.05              1006   202 .01
    12   301-.05              31   301-0.2                32   301 0.15
    51   301 -.05           1002   301-.05              1006   301 0.01
KER  1     2                  -1
   992   992  10000000.      992  1002 -10000000.        996   996  2000.
   996  1006 -2000.         1002  1002  10000000.       1006  1006  2000.
  RB                         10.0 E6
00000000                                                      3.85 E6
00000000
       1    -5.0   25.0
       2     5.0   17.5
       3    15.0   10.0
       4    25.0    2.5
       5    35.0   -5.0


1  2 21      1        2        .5                       1.5                    RB
1  3 21      2        3        .5                       1.5                    RB
1  4 21      3        4        .5                       1.5                    RB
1  5 21      4        5        .5                       1.5                    RB
    12     31     32     51  1002  1006
    12     31     32     51  1002  1006
FOC  1     4                   1
    12   101-5.0      E-2     31   101-20.0      E-2      32   101 15.      E-2
    51   101-5.0      E-2   1002   101 -5.0      E-2    1006   101  1.0     E-2
    12   102   -1.0           21   201  -30.              22   201  -40.
    12   301  -1.0            21   301  -30.              22   301  -40.
 DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GEN WEIGHT MATRICES
 ROW AND COLUMN CORESPOND TO MODE, LOAD, OR REACTION DISPLACEMENT OR ACC
 EXCEPT DDC  1 ROW LEADING DIGITS CORRESPOND TO GRIDPOINT AND FINAL
 DIGIT TO THE DEGREE OF FREEDOM AT THE GRIDPOINT
 REACTION NOT DISPLACEMENT  AT RESTRAINT
 INTERNAL FORCE FOR ELEMENT -X- (RDC X )
 ROW LEADING DIGITS CORRESPOND TO GRIDPOINT,FINAL DIGIT
 1    AXIAL FORCE                          4    TORQUE
 8    SHEAR ALONG X2                       5    MOMENT ABOUT X2
 7    SHEAR ALONG X3                       9    MOMENT ABOUT X3
                   COL CORRESPONDS TO MODE,LOAD,OR REACT. DISP. OR ACC.
```

Output

0.0	-0		-0	-0		-0	CHEX	-0		-0	1
1.0	9001	WTR	1	9002	ARC	1	READ	9003	KER	1	-100
2.0	9004	KER	2	10001	SCR	2	BILD	-0		-0	-405
3.0	9003	KER	1	-0		-0	ADDS	12001	KTR	1	500
4.0	12001	KTR	1	9002	ARC	1	CHOL	12002	ATC	1	51001
4.1	-0		-0	11001	KUR	1	CONT	-0		-0	-0
5.0	-0		-0	-0		-0	ERRS	-0		-0	-0
5.1	11001	KUR	1	-0		-0	CHOL	9001	RBC999		92000
5.2	9001	RBC999		-0		-0	INKS	-0		-0	-0
5.3	-0		-0	-0		-0	STOP	-0		-0	-0
6.0	9001	WTR	1	-0		-0	CHOL	-0		-0	1
6.1	-0		-0	11002	WUR	1	CONT	-0		-0	-0
7.0	11002	WUR	1	-0		-0	FLIP	12003	WLC	1	-0
8.0	11001	KUR	1	12003	WLC	1	CHOL	9002	TEC	1	90000
9.0	11002	WUR	1	9002	TEC	1	MULT	12003	DCC	1	-0
10.0	12003	DCC	1	-0		-0	DECO	11003	DDC	1	-0
11.0	11003	DDC	1	11004	VDC	1	ROOT	-0		-0	-6
12.0	11004	VDC	1	-0		-0	CODE	9002	VCC	1	-0
13.0	11002	WUR	1	9002	VCC	1	CHCL	12003	DWC	1	89000
14.0	12002	ATC	1	12003	DWC	1	ADDS	11005	AUC	2	-0
15.0	9001	WTR	1	11005	AUC	2	MULT	9002	DFC	1	-0
16.0	-0	FOC	1	-0		-0	READ	-0		-0	-100
17.0	-0	FOC	1	9002	DFC	1	ADDS	12001	FOC	2	-0
18.0	11001	KUR	1	12001	FOC	2	CHOL	9002	DTC	1	91000
18.1	12002	DDC	1	-0		-0	CONT	-0		-0	-0
19.0	12002	DDC	1	-0		-0	COLS	11006	GCC	1	-0
20.0	12001	FOC	2	-0		-0	FLIP	-0	FOR	2	-0
21.0	-0	FOR	2	12002	DDC	1	MULT	11007	GCC	2	-0
22.0	9001	WTR	1	9002	DTC	1	MULT	12003	FOC	3	-0
23.0	12003	FOC	3	-0		-0	FLIP	-0	FOR	3	-0
24.0	-0	FOR	3	9002	DTC	1	MULT	11008	GCC	3	-0
25.0	11006	GCC	1	-0		-0	DECO	12002	DDC	1	300
26.0	12002	DDC	1	-0		-0	INKS	-0		-0	305
27.0	9002	DTC	1	-0		-0	FILL	-0		-0	-0
28.0	10001	SCR	2	-0	DTC	1	MULT	11009	RCC	2	400
29.0	11009	RCC	2	-0		-0	DECO	10001	RDC	2	400
30.0	10001	RDC	2	-0		-0	INKS	-0		-0	406
31.0	-0		-0	-0		-0	HALT	-0		-0	-0

ALTERNATE INPUT HAS BEEN REQUESTED. A LISTING OF THE GRIDPOINT COORDINATE TABLE FOLLOWS

GRIDPOINT	X	Y	Z
1	-5.0	25.0	
2	5.0	17.5	
3	15.0	10.0	
4	25.0	2.5	
5	35.0	-5.0	

M A T R I X   I N P U T   D A T A

WTR	1	4	-0	-0	-1	-0	-0	-0		
		11	11	0.1500000E 01	21	21	0.3000000E 01	22	22	0.3000000E 01
		41	41	0.3000000E 01	42	42	0.3000000E 01	52	52	0.1500000E 01
		992	992	0.2000000E 01	992	996	0.1000000E 02	996	992	0.1000000F 02
		996	996	0.6200000E 02	00	00	0.0000000E-38	00	00	0.0000000E-38
ARC	1	4	-0	-0	1	-0	-0	-0		
		12	202	-0.5000000E-01	31	202	-0.2000000E 00	32	202	0.1500000E 00
		51	202	-0.5000000E-01	1002	202	-0.5000000E-01	1006	202	0.1000000E-01
		12	301	-0.5000000E-01	31	301	-0.2000000E 00	32	301	0.1500000E 00
		51	301	-0.5000000E-01	1002	301	-0.5000000E-01	1006	301	0.1000000E-01
KER	1	2	-0	-0	-1	-0	-0	-0		
		992	992	0.1000000E 08	992	1002	-0.1000000E 08	996	996	0.2000000E 04
		996	1006	-0.2000000F 04	1002	1002	0.1000000E 08	1006	1006	0.2000000E 04

M A T E R I A L   T A B L E

F I E L D

1	2	3	4	5	6	7	8	9
RB	-0.0000E-38	-0.0000E-38	0.1000E 08	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38
	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	0.3850E 07	

E L E M E N T   D A T A

F I E L D

1	2	3	4	5	6	7	8	9	10	11	12	13
1	2	21	1.	2.	-0.	0.5000E 00	-0.0000E-38	-0.0000E-38	0.1500E 01	-0.0000E-38	-0.0000E-38	RB
9	2	21	-0.5000E 01	0.2500E 02	-0.0000E-38	0.5000E 01	0.1750E 02	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	

ELEMENT 2 HAS ONLY 2 FLEXIBILITIES DUE TO 2 ZERO GEOMETRIC PROPERTIES.

1	3	21	2.	3.	-0.	0.5000E 00	-0.0000E-38	-0.0000E-38	0.1500E 01	-0.0000E-38	-0.0000E-38	RB.
9	3	21	0.5000E 01	0.1750E 02	-0.0000E-38	0.1500E 02	0.1000E 02	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	

ELEMENT 3 HAS ONLY 2 FLEXIBILITIES DUE TO 2 ZERO GEOMETRIC PROPERTIES.

1	4	21	3.	4.	-0.	0.5000E 00	-0.0000E-38	-0.0000E-38	0.1500E 01	-0.0000E-38	-0.0000E-38	RB.
9	4	21	0.1500E 02	0.1000E 02	-0.0000E-38	0.2500E 02	0.2500E 01	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	

ELEMENT 4 HAS ONLY 2 FLEXIBILITIES DUE TO 2 ZERO GEOMETRIC PROPERTIES.

1	5	21	4.	5.	-0.	0.5000E 00	-0.0000E-38	-0.0000E-38	0.1500E 01	-0.0000E-38	-0.0000E-38	RB.
9	5	21	0.2500E 02	0.2500E 01	-0.0000E-38	0.3500E 02	-0.5000E 01	-0.0000E-38	-0.0000E-38	-0.0000E-38	-0.0000E-38	

ELEMENT 5 HAS ONLY 2 FLEXIBILITIES DUE TO 2 ZERO GEOMETRIC PROPERTIES.

MATRIX KTR 1 IS OF ORDER 19 AND HAS MAXIMUM WAVEFRONT OF 6 , A DIAGONAL 0.0000E-38 OF INITIAL VALUE
AND AN UNPRESCRIBED DIAGONAL 0.3754E 00 OF INITIAL VALUE

OPERATION CHOL CALLED AT TIME 212146
12 31 32 51 1002 1006 00 00 00 00 00 00 PRESCRIBED

MATRIX WTR 1 IS OF ORDER 8 AND HAS MAXIMUM WAVEFRONT OF 2 , A DIAGONAL 0.1935E 00 OF INITIAL VALUE
AND AN UNPRESCRIBED DIAGONAL 0.1935E 00 OF INITIAL VALUE

ROOTS OF MATRIX DDC 1

NO.	ROOT=R*R	1/R*R	R	1/R	R/2PI	1/(2PI*R)	SQ.RT.G/(2PI*R)
1	3.1000E-02	3.2258E 01	1.7607E-01	5.6796E 00	2.8022E-02	9.0394E-01	1.7769E 01
2	7.1628E-05	1.3961E 04	8.4633E-03	1.1816E 02	1.3470E-03	1.8805E 01	3.6966E 02
3	3.3240E-05	3.0084E 04	5.7654E-03	1.7345E 02	9.1760E-04	2.7605E 01	5.4265E 02
4	2.0511E-05	4.8754E 04	4.5289E-03	2.2080E 02	7.2080E-04	3.5142E 01	6.9080E 02
5	1.5714E-05	6.3639E 04	3.9640E-03	2.5227E 02	6.3090E-04	4.0150E 01	7.8924E 02
6	3.0846E-06	3.2419E 05	1.7563E-03	5.6937E 02	2.7953E-04	9.0619E 01	1.7813E 03

M A T R I X   I N P U T   D A T A

FOC	1	4	-0	-0	1	-0	-0	-0	
		12	101	-0.5000000E-01	31	101	-0.2000000E 00	32	101	0.1500000E 00
		51	101	-0.5000000E-01	1002	101	-0.5000000E-01	1006	101	0.1000000E-01
		12	102	-0.1000000E 01	21	201	-0.3000000E 02	22	201	-0.4000000E 02
		12	301	-0.1000000E 01	21	301	-0.3000000E 02	22	301	-0.4000000E 02

DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GEN WEIGHT MATRICES

ROW AND COLUMN CORESPOND TO MODE, LOAD, OR REACTION DISPLACEMENT OR ACC

EXCEPT DDC 1 ROW LEADING DIGITS CORRESPOND TO GRIDPOINT AND FINAL

DIGIT TO THE DEGREE OF FREEDOM AT THE GRIDPOINT

REACTION NOT DISPLACEMENT AT RESTRAINT

DDC 1 DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GEN WEIGHT MATRICES

COLS ** ROWS OF THE TRANSPOSED MATRIX **

	11	12	16	21	22	26	31	32
01	0.3083E-22	0.5235E-17	-0.5597E-22	-0.2535E-21	-0.3522E-21	0.3376E-23	-0.1644E-16	0.3311E-16
02	0.3517E-05	-0.7879E 00	0.3006E-05	0.1736E-04	0.2068E-04	-0.4707E-07	-0.1460E 01	0.1232E 01
03	-0.3566E-05	0.6155E 00	-0.1601E-05	-0.9422E-05	-0.9883E-05	0.4607E-06	0.2371E 00	0.1699E 01
04	0.1949E-05	-0.2134E 00	0.2440E-06	0.2186E-05	0.1324E-05	-0.1998E-06	-0.2258E 01	0.7970E 00
05	0.1004E-04	-0.5604E 00	-0.8390E-06	0.3196E-05	-0.4532E-05	-0.1782E-06	-0.1400E 01	0.1446E 01
06	-0.1729E-08	0.1938E-02	-0.2209E-08	-0.9999E-09	-0.6676E-08	0.3241E-08	-0.4967E 00	0.3981E 00
101	-0.3500E 00	0.1831E-03	0.1000E-01	-0.2750E 00	0.5000E-01	0.1000E-01	-0.4901E-02	0.4395E-02
102	-0.7417E 00	-0.2213E 04	0.6209E-01	-0.3017E 00	-0.4078E 00	0.5287E-01	0.2950E 04	0.2213E 04
201	-0.9511E-04	0.2536E 02	-0.9522E-04	-0.5146E-03	-0.6228E-03	0.1046E-04	0.3785E 02	0.1464E 02
202	-0.7361E-05	0.6380E 00	-0.8550E-06	-0.9624E-05	-0.6361E-05	0.1629E-06	0.2724E 01	-0.2063E 01
301	-0.7418E 00	-0.2187E 04	0.6200E-01	-0.3022E 00	-0.4084E 00	0.5288E-01	0.2991E 04	0.2225E 04

	36	41	42	46	51	52	56	992
01	0.3803E-22	0.1656E-21	0.9734E-22	-0.8824E-23	0.1994E-17	-0.1471E-21	-0.3098E-22	0.1270E-06
02	-0.3324E-05	-0.1548E-04	-0.2611E-04	-0.3692E-06	0.1307E 01	-0.1037E-04	0.2810E-05	-0.1032E-23
03	0.2713E-06	-0.3134E-05	-0.1031E-04	-0.7236E-06	0.1057E 01	-0.1089E-04	0.5324E-06	-0.4217E-21
04	0.2745E-06	0.6820E-05	0.1165E-05	-0.2616E-06	0.7981E 00	-0.1296E-04	-0.1716E-05	0.1078E-23
05	0.1056E-06	-0.7549E-06	-0.4948E-06	-0.3790E-07	-0.2446E-01	0.7793E-06	0.1956E-06	0.1417E-22
06	-0.1050E-07	0.1253E-05	-0.9753E-06	0.3332E-07	-0.7198E 00	0.1141E-05	0.9634E-07	-0.3750E-2.
101	0.1000E-01	-0.1250E 00	0.2500E 00	0.1000E-01	0.0000E-38	0.3500E 00	0.1000E-01	-0.5000E-01
102	0.2522E-01	0.6915E-01	0.1020E 00	-0.2444E-02	-0.2950E 04	0.1967E-01	-0.1166E-01	0.0000E-38
201	0.6708E-04	0.1840E-03	0.2714E-03	-0.6502E-05	-0.7849E 01	0.5233E-04	-0.3103E-04	0.0000E-38
202	0.1263E-05	0.3823E-05	0.1287E-04	0.4110E-06	-0.9993E 00	0.1241E-04	-0.5243E-06	0.0000E-38
301	0.2528E-01	0.6934E-01	0.1023E 00	-0.2450E-02	-0.2959E 04	0.1973E-01	-0.1170E-01	0.0000E-38

	996	1002	1006
01	0.3937E-02	-0.1270E 01	-0.7874E 01
02	0.5708E-24	0.1032E-16	-0.1142E-20
03	0.1241E-21	0.4217E-14	-0.2482E-18
04	-0.1489E-24	-0.1078E-16	0.2978E-21
05	-0.2197E-23	-0.1417E-15	0.4394E-20
06	0.6247E-23	0.3750E-15	-0.1249E-19
101	0.1000E-01	0.0000E-38	0.0000E-38
102	0.0000E-38	0.0000E-38	0.0000E-38
201	0.0000E-38	0.0000E-38	0.0000E-38
202	0.6000E-04	0.0000E-38	-0.1200E 00
301	0.6000E-04	0.0000E-38	-0.1200E 00

THIS COMPLETES PRINTOUT OF MATRIX 12002 DDC 1.

DDC 2 DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GEN WEIGHT MATRICES

COLS *** ROWS OF THE TRANSPOSED MATRIX ***

	01	02	03	04	05	06	101	102
01	0.3100E-01	-0.6658E-21	0.9011E-21	0.1765E-21	0.1363E-21	0.3183E-22	-0.1524E-01	-0.5235E-17
02	-0.6658E-21	0.7163E-04	0.1535E-11	0.0000E-38	-0.2416E-12	0.3979E-12	0.4508E 00	0.7879E 00
03	0.9011E-21	0.2103E-11	0.3324E-04	0.9095E-12	0.2913E-12	-0.1705E-12	0.1238E 00	-0.6155E 00
04	0.1765E-21	0.1990E-12	0.9663E-12	0.2051E-04	-0.3553E-12	0.1137E-12	0.5419E 00	0.2134E 00
05	0.1363E-21	-0.4974E-13	0.4370E-12	-0.3340E-12	0.1571E-04	-0.3197E-13	0.5262E 00	0.5604E 00
06	0.3183E-22	-0.4796E-12	-0.9948E-13	0.2274E-12	-0.3819E-13	0.3085E-05	0.1949E 00	-0.1938E-02
101	0.1524E-01	-0.4508E 00	-0.1238E 00	-0.5419E 00	-0.5262E 00	-0.1949E 00	0.1630E-02	-0.1831E-03
102	0.5235E-17	-0.7879E 00	0.6155E 00	-0.2134E 00	-0.5604E 00	0.1938E-02	-0.6294E-04	0.2213E 04
201	0.2169E-19	-0.1348E-02	0.6780E-03	-0.1185E-03	0.8541E-04	0.2970E-06	-0.6250E 01	-0.2536E 02
202	0.4724E-03	-0.3229E-04	-0.4114E-05	-0.1111E-04	-0.8268E-05	-0.6013E-06	-0.8374E 00	-0.6380E 00
301	0.4724E-03	-0.7893E 00	0.6161E 00	-0.2136E 00	-0.5603E 00	0.1938E-02	-0.7087E 01	0.2187E 04

	201	202	301
01	0.2169E-19	0.4724E-03	0.4724E-03
02	-0.1348E-02	-0.3229E-04	0.7865E 00
03	0.6780E-03	-0.4114E-05	-0.6148E 00
04	-0.1185E-03	-0.1111E-04	0.2133E 00
05	0.8541E-04	-0.8268E-05	0.5605E 00
06	0.2970E-06	-0.6013E-06	-0.1939E-02
101	0.6250E 01	0.8374E 00	0.7087E 01
102	0.2536E 02	0.6380E 00	0.2239E 04
201	0.4035E-01	0.5432E-03	-0.2532E 02
202	0.5432E-03	0.3278E-04	-0.6375E 00
301	0.2540E 02	0.6386E 00	0.2213E 04

THIS COMPLETES PRINTOUT OF MATRIX 12003 DDC 2.

DDC 3 DDC 1,2,3 = DISPLACEMENT, GENERALIZED STIFFNESS AND GEN WEIGHT MATRICES

COLS *** ROWS OF THE TRANSPOSED MATRIX ***

	01	02	03	04	05	06	101	102
01	0.9610E-03	0.5077E-25	0.1370E-22	0.9706E-26	0.2364E-25	0.4872E-25	0.4724E-03	0.6858E-21
02	0.5077E-25	0.5131E-08	0.5031E-16	0.2429E-16	-0.7698E-17	-0.3448E-16	-0.3229E-04	-0.5644E-04
03	0.1370E-22	0.5031E-16	0.1105E-08	0.2429E-16	0.1008E-16	-0.4554E-17	-0.4114E-05	0.2046E-04
04	0.9706E-26	0.2255E-16	0.2255E-16	0.4207E-09	-0.9975E-17	0.8023E-17	-0.1111E-04	-0.4378E-05
05	0.2364E-25	-0.4228E-17	0.1019E-16	-0.1041E-16	0.2469E-09	-0.1152E-17	-0.8268E-05	-0.8806E-05
06	0.4872E-25	-0.3534E-16	-0.4770E-17	0.8240E-17	-0.1152E-17	0.9515E-11	-0.6013E-06	0.5979E-08
101	0.4724E-03	-0.3229E-04	-0.4114E-05	-0.1111E-04	-0.8268E-05	-0.6013E-06	0.8374E 00	0.6380E 00
102	0.6858E-21	-0.5644E-04	0.2046E-04	-0.4378E-05	-0.8806E-05	0.5979E-08	0.6380E 00	0.1643E 01
201	0.1204E-23	-0.9656E-07	0.2254E-07	-0.2431E-08	0.1342E-08	0.9168E-12	0.5432E-03	0.1456E-02
202	0.1465E-04	-0.2313E-08	-0.1368E-09	-0.2280E-09	-0.1299E-09	-0.1855E-11	0.3278E-04	0.2978E-04
301	0.1465E-04	-0.5653E-04	0.2048E-04	-0.4380E-05	-0.8805E-05	0.5978E-08	0.6386E 00	0.1645E 01

	201	202	301
01	0.1204E-23	0.1465E-04	0.1465E-04
02	-0.9656E-07	-0.2313E-08	-0.5653E-04
03	0.2254E-07	-0.1368E-09	0.2048E-04
04	-0.2431E-08	-0.2280E-09	-0.4380E-05
05	0.1342E-08	-0.1299E-09	-0.8805E-05
06	0.9168E-12	-0.1855E-11	0.5978E-08
101	0.5432E-03	0.3278E-04	0.6386E 00
102	0.1456E-02	0.2978E-04	0.1645E 01
201	0.2298E-05	0.4135E-07	0.1459E-02
202	0.4135E-07	0.2245E-06	0.3005E-04
301	0.1459E-02	0.3005E-04	0.1646E 01

THIS COMPLETES PRINTOUT OF MATRIX 12004 DDC 3.

INTERNAL FORCE FOR ELEMENT -X- (RDC X)

ROW LEADING DIGITS CORRESPOND TO GRIDPOINT,FINAL DIGIT

1	AXIAL FORCE	4	TORQUE
8	SHEAR ALONG X2	5	MOMENT ABOUT X2
7	SHEAR ALONG X3	9	MOMENT ABOUT X3

COL CORRESPONDS TO MODE,LOAD,OR REACT. DISP. OR ACC.

RDC 2 INTERNAL FORCE FOR ELEMENT -X- (RDC X)

COLS *** ROWS OF THE TRANSPOSED MATRIX ***

	11	18	19	29
01	0.6468E-17	-0.1139E-16	-0.8582E-23	-0.1424E-15
02	0.5317E 00	0.5861E 00	0.4470E-06	0.7327E 01
03	-0.4980E 00	-0.3958E 00	-0.1788E-06	-0.4948E 01
04	0.2421E 00	0.8522E-01	0.5215E-07	0.1065E 01
05	0.1103E 01	-0.1269E 00	-0.1080E-06	-0.1586E 01
06	-0.1836E-02	-0.1046E-02	-0.1164E-09	-0.1308E-01
101	0.3906E-02	-0.7324E-03	-0.2441E-02	-0.5371E-02
102	0.1328E 04	0.1770E 04	-0.2930E-02	0.2213E 05
201	-0.1522E 02	-0.2029E 02	-0.7868E-05	-0.2536E 03
202	-0.8028E 00	-0.1954E 00	-0.1900E-06	-0.2443E 01
301	0.1312E 04	0.1750E 04	0.2930E-02	0.2187E 05

THIS COMPLETES PRINTOUT OF MATRIX 10001 RDC 2.

RDC 3 INTERNAL FORCE FOR ELEMENT -X- (RDC X)

COLS	****** ROWS OF THE TRANSPOSED MATRIX ******			
	21	28	29	39
01	0.3398E-17	0.1614E-16	0.1424E-15	0.5927E-16
02	0.5939E 00	-0.5431E 00	-0.7327E 01	0.5378E 00
03	-0.6431E 00	0.8280E 00	0.4948E 01	0.5402E 01
04	0.3817E 00	-0.2615E 00	-0.1065E 01	-0.2204E 01
05	0.2111E 01	0.1993E 00	0.1586E 01	0.9048E 00
06	0.1282E-02	0.4732E-02	0.1308E-01	0.4607E-01
101	0.2441E-02	0.6104E-04	0.2441E-02	0.0000E-38
102	0.1328E 04	0.1770E 04	-0.2213E 05	0.4425E 05
201	-0.1522E 02	0.2971E 02	0.2536E 03	0.1177E 03
202	-0.1553E 01	0.1796E 00	0.2443E 01	-0.1985E 00
301	0.1311E 04	0.1800E 04	-0.2187E 05	0.4437E 05

THIS COMPLETES PRINTOUT OF MATRIX 10002 RDC 3.

RDC 4 INTERNAL FORCE FOR ELEMENT -X- (RDC X)

COLS	****** ROWS OF THE TRANSPOSED MATRIX ******			
	31	38	39	49
01	-0.2963E-16	-0.4882E-18	-0.5927E-16	0.5317E-16
02	-0.1313E 01	-0.6533E 00	-0.5378E 00	-0.7629E 01
03	-0.1473E 01	-0.6733E 00	-0.5402E 01	-0.3014E 01
04	-0.1903E 01	0.4555E 00	0.2204E 01	0.3490E 01
05	0.1228E 00	-0.1172E 00	-0.9048E 00	-0.5604E 00
06	-0.6349E 00	-0.1579E-01	-0.4607E-01	-0.1512E 00
101	-0.3906E-02	0.2441E-03	0.9766E-03	0.4883E-02
102	0.2360E 04	-0.1770E 04	-0.4425E 05	0.2213E 05
201	0.6279E 01	-0.4709E 01	-0.1177E 03	0.5887E 02
202	0.1864E 01	0.1954E 00	0.1985E 00	0.2245E 01
301	0.2368E 04	-0.1775E 04	-0.4437E 05	0.2219E 05

THIS COMPLETES PRINTOUT OF MATRIX 10003 RDC 4.

RDC 5 INTERNAL FORCE FOR ELEMENT -X- (RDC X)

COLS	****** ROWS OF THE TRANSPOSED MATRIX ******			
	41	48	49	59
01	-0.5683E-17	-0.4254E-17	-0.5317E-16	0.0000E-38
02	-0.1176E 01	0.6103E 00	0.7629E 01	0.1192E-06
03	-0.1140E 01	0.2411E 00	0.3014E 01	0.8941E-07
04	-0.1207E 01	-0.2792E 00	-0.3490E 01	0.0000E-38
05	0.6420E-01	0.4483E-01	0.5604E 00	0.0000E-38
06	0.9088E 00	0.1210E-01	0.1512E 00	-0.1118E-07
101	0.0000E-38	-0.6104E-04	0.2441E-03	0.4883E-03
102	0.2360E 04	-0.1770E 04	-0.2213E 05	-0.4883E-03
201	0.6279E 01	-0.4709E 01	-0.5887E 02	0.0000E-38
202	0.1114E 01	-0.1796E 00	-0.2245E 01	0.0000E-38
301	0.2368E 04	-0.1775E 04	-0.2219E 05	0.4883E-03

THIS COMPLETES PRINTOUT OF MATRIX 10004 RDC 5.

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